

When Financial Economics influences Physics: The Role of Econophysics

2018 American Economic Association Annual Meeting
Philadelphia PA January 05 — 07

First draft (do not cite without permission)

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Abstract

This paper aims at discussing the unexpected influence of Financial economics on Physics. The rise of Econophysics, a fundamentally new approach in finance, suggests that the influence between the two disciplines becomes less unilateral than in the past. Methodological debates emerging in Econophysics led physicists to acknowledge that dealing with financial complex systems contributed to a better modelling of their field. The approach of econophysicists suggests that physicists might try to conceptualize physical phenomena by integrating elements they faced with in Financial economics, and more generally in Economics. Surprisingly, many of econophysicists' argumentations have some methodological similarities with practices used in Financial economics. This paper analyzes the influence of Financial economics and Economics on Physics by discussing three examples: out of equilibrium processes, signal detection and information. It investigates and illustrates what are the methodological changes generated by Econophysics that explain this new influence, and what is the role of finance. This paper sheds new light on the traditional distinction between “hard sciences” (like Physics) and “soft sciences” (like Economics) and the specific situation of Financial economics in this movement.

Keywords: Econophysics, out of equilibrium processes, information, random matrix, El Farol bar, Minority game, analogies
JEL: G00, B26

1. Introduction

The influence of Physics on Financial economics, and more generally on Economics is an indisputable fact. A number of writers have extensively highlighted contributions of Physics to the development of Economics and mathematical economics (Mirowski 1989, Ingrao and Israel 1990, Le Gall 2002, Maas 2005, Ménard 1981, Schabas 1990, Boumans 2007, 2004, Morgan and Morrison 1999, Poitras and Heaney 2015). Financial economics, and more generally finance, is also subject to the influence of Physics (Jovanovic and Schinckus 2013, 2017, Jovanovic and Le Gall 2001, Poitras 2000, 2006, Sornette 2014). We can mention, among a large variety of examples, the concept of (mechanical) equilibrium, the random walk theory and its variant of the geometric Brownian motion, the stable Lévy distributions, etc. However, the rise of a fundamentally new approach in the 1990s called Econophysics (Mantegna and Stanley 2000, Jovanovic and Schinckus 2017) suggests that the influence between the two disciplines becomes less unilateral than in the past, and surprisingly Financial economics has an unexpected influence on Physics.

The term *econophysics* generally refers to the extension of methods and tools traditionally introduced and developed in the field of statistical and theoretical physics to the study of problems commonly considered to fall within the sphere of Economics, and particularly problems in finance. Over the past two decades, Econophysics has carved out a place in the scientific analysis of financial markets, providing new theoretical models, methods, and results (Bouchaud, Mezard, and Potters 2002, Potters and Bouchaud 2003, McCauley 2009, Gabaix 2009, Lux 2009, McCauley, Gunaratne, and Bassler 2007, Sornette 2014). The framework that econophysicists have developed describes the evolution of financial markets in a way very different from that used by the current standard financial models. Today, although less visible than Financial economics, Econophysics influences financial markets and practices (Jovanovic and Schinckus 2017, chap. 5). Many “quants” (quantitativists) trained in Statistical physics have carried their tools and methodology into the financial world. According to several trading-room managers and directors, econophysicists’ phenomenological approach has modified the practices and methods of analyzing financial data. Hitherto, these practical changes have concerned certain domains of finance: hedging, portfolio management, financial crash predictions, and software dedicated to finance (Jovanovic and Schinckus 2017, chap. 5, Bouchaud and Challet 2014, Sornette 2013, Sornette and Cauwels 2015, Mantegna and Stanley 2000, Casey 2013).

Econophysics is an example of hybrid discipline. The end of the past century and the beginning of the present one have seen the development of several research areas covering a field between the boundaries of traditional disciplines. Examples are bioinformatics, system biology, cognitive science, network science to cite only a few of them. In this type of disciplines, scholars with a background from two or more well established disciplines start to interact and develop the social and cultural environment needed to develop a new scientific research area characterized by a set of scientific problems, methods, tools and scientific practice. The specificity of each of the two disciplines of Physics and Economics setting the boundary of Econophysics implies an influence from one discipline (or sub-discipline) to the other and vice versa. Precisely, the econophysicists’ way of dealing with large quantity of data describing economic, social and financial systems offers a telling example of empirical analyses

performed in the absence of micro-founded theories. Today this type of analysis is more common than twenty years ago among scholars of Economics and finance as it is testified by the number of papers dealing with large set of data accepted in academic leading journals of Economics and finance.

From a financial economist's viewpoint, Econophysics aims to provide models that reproduce the statistical behaviors of stock price or return variations, including their extreme values, and then to apply these models to the study of financial products and strategies, such as options pricing, portfolio optimization, trading decisions of individual investors, or stock market crashes. The first use in print of the neologism econophysics came in a 1996 article by Stanley et al. (1996). Stanley was also the first to use the word econophysics in a public occasion¹. The first research community of econophysicists formed in a series of Workshops entirely dedicated to the analyses and modelling of economic and financial systems with methods and tools of statistical and theoretical physics. These were the workshops organized in Budapest (21-27 July 1997), Rome (12-13 March 1998), and Palermo (28-30 September 1998). However, following Kutner and Grech (2008), we can trace the informal birth of the movement to a paper published by Mantegna (1991) that studied the evolution of financial index of the Milan Stock Exchange in terms of Lévy walks. This birth finds its origins in some changes that have occurred in the 1970s and the 1980s in Statistical physics, particularly new developments in the renormalization group theory (Lesne and Laguës 2012, Lesne 1998, Stanley 1999, Jovanovic and Schinckus 2017, chap. 3)², and on financial markets, particularly new financial data and the digitalization of financial markets' operations (Jovanovic and Schinckus 2017, chap. 3). Nowadays, Econophysics provides results to most of the areas of capital markets (market microstructure, CAPM, option pricing, price variations, financial cracks, portfolio selection, etc.)³.

Econophysics is a fundamentally new approach, although some roots can be traced back to Mandelbrot's work, which had Fama (Fama 1963b, a) inspired and other financial economists like Fama and Roll (1968, 1971), Blattberg and Sargent (1971), Teichmoeller (1971), Clark (1973) and Brenner (1974)⁴. Econophysicists are not economists or finance taking their inspiration from the work of physicists to develop their discipline, as it has been seen repeatedly in the history of Economics. This time, it is physicists who are going beyond the boundaries of their discipline, using their own methods and models to study various problems thrown up by Economics. Such a specific movement has some interesting singularities.

¹ Actually, Stanley was the first scholar using the term "econophysics" during a conference on the "Physics of Complex Systems" organized in Kolkata (India) in 1995 (Chakrabarti and Chakraborti 2010).

² "The development of [the renormalization group] technique undoubtedly represents the single most significant advance in the theory of critical phenomena and one of the most significant in theoretical physics generally" since the 1970s (Alastair and Wallace 1989, 237).

³ Capital markets and investments are not the only areas concerned by Econophysics. We can also mention money, macroeconomics, etc.

⁴ Depict these roots, Econophysics must be separated from Mandelbrot's project (Jovanovic and Schinckus 2017). While Mandelbrot and econophysicists arrive at the same result —modeling stock price variations using Lévy stable processes—, Mandelbrot starts his analysis from the stability of stochastic process and the generalized central-limit theorem, explaining why he starts systematically from a stable Lévy distribution, in contrast, econophysicists' starting point is critical phenomena and the results obtained from renormalization group methods, explaining why they start systematically from power-law distributions.

As explained in Jovanovic and Schinckus (2017, chap. 4), the singular institutional position of Econophysics —outside Financial economics and in the shadow of Physics— has structured exchanges between econophysicists and financial economists. While it is not hard to understand that this disciplinary structure makes dialogue difficult between Financial economics and Econophysics, it has provided a surprisingly fruitful context for scientific innovations. Precisely, from the observations of economic phenomena, and particularly finance phenomena, Econophysics has given the opportunities to develop new hypothesis, models and methods outside Physics, and then bring them back to Physics. Some examples of such developments are the way econophysicists contribute to the modelling of out of equilibrium processes, signal detection in multivariate systems and information process and aggregation in multi-agents physical systems, as the next section explains.

Such examples suggest that the links between Physics and finance has recently changed. It is worth mentioning that econophysicists' perspective has also changed, defending nowadays a “mutual fertilization”⁵ between Physics and Financial economics rather than an unidirectional influence of physics on finance, as it was common in the past (Sornette 2014, Sinha, Chakrabarti, and Mitra 2016). This article argues that due to its use of financial data newly available, its methods, its methodology and also the new challenges it has faced to by studying financial markets, Econophysics has reversed the traditional relation between Physics and Economics. Section 2 will analyze how Econophysics' research in finance has changed the modelling of equilibrium in Physics. Section 3 will explain the key role of finance in Physics nowadays and the specific methods used in Econophysics. Section 4 will conclude about the new influence of Financial economics on Physics.

2. Three examples of the influence of Finance on Physics

This section uses three examples in order to show the influence of finance on Physics: the case out of equilibrium processes, the signal detection in a multivariate characterization of a complex system, and the elucidation of information role in a complex adaptive system modeled with Statistical physics tools.

2. 1. The case of out of equilibrium processes

Physics and Financial economics are both empirical disciplines but they did not have developed the same way of dealing with data. Such situation generated unexpected contributions on both side.

The mainstream approach of Physics is built upon the observation that some quantities are conserved during the time evolution of the studied system. Basic examples are energy, mass, charge, momentum, etc. In other words, major successes of Physics are observed when conservation principles are present in the investigated systems. Another characterizing aspect associated with Physics is that experimental verification of models' predictions is performed in highly controlled experimental settings. Laboratory experiments in Physics are planned in a way to minimize the

⁵ This expression is borrowed from Sornette (2014, 1).

sources of uncertainty due to uncontrolled events. Experimental set-ups are designed to focus on the studied phenomena. In other words, the phenomena of interest are singled out and the role of any other potential influence is limited as much as possible in the setting. This type of experimental settings implies that the so-called signal to noise ratio is usually high and often not requiring highly sophisticated procedures of statistical validation of the experimental results obtained. It is precisely on this point that finance, and more generally disciplines dealing with open systems whose time evolution is not characterized by conservation laws, suggested new paths in Physics.

In Financial economics, basic concepts most commonly used to build up models are (i) the paradigm of the absence of arbitrage opportunities, which is linked with the efficient market hypothesis, (ii) rational agent, able to subsume all heterogeneities that are present in a real system, that is taking rational decisions by being able to process accurately all information available, and (iii) process of optimization of rational agents with respect to a given utility function. The presence of these basic concepts it is often summarized in the requirement that models describing economic and financial systems needs to be correctly “micro-founded”. This micro-founded perspective developed by financial economists refers to the necessity to explain phenomena in terms of agents’ behaviors. However, the cultural background of Physics of econophysicists and their freedom from the need of a micro-founded approach has motivated them to perform empirical analyses also in the absence of a micro-founded theory and/or in the absence of a a-priori characterization of the stochastic process of interest. As Schinckus (2014) explained, econophysicists assume that heterogeneous micro-interactions are too complex to be captured through the action of a representative agent so that they do not provide a framework compatible with the classical idea of reduction⁶. When they refer to agents, econophysicists implicitly assume the agents’ behavior is heterogeneous, and that social interactions of heterogeneous agents can be associated with the emergence of global behaviors that are not crucially dependent on the individual choices of economic agents. This phenomenological methodology, reducing the agents’ heterogeneity to a collective activity on the macro-scale preserves, is in line with a micro-indeterminism inducing, by coarse-graining, a macro-determinism. This “coarse-graining situation” is well-known in hard science but not so common in social sciences in which agents are endowed with intentions. The impossibility to define the high number of microscopic configurations for individuals implicitly refers to what we call “the multiple realizability argument” – such approach can be useful for the characterization of “stylized facts” (persistent macro-regularities which cannot be described in terms of microeconomic theory) – among the most studied stylized facts, one can mention: heavy tails of financial distributions, volatility clustering, volume/volatility correlation, absence of autocorrelation in asset return dynamics, emergent phenomena in heterogeneous systems, etc. (Cont 2001, Buchanan 2012).

Interestingly, by studying socio-economic systems, some (econo)physicists also start to provide micro-founded framework to their works as witnessed by the increasing number of agent-based modelling in Physics. Methodologically, this modelling takes the form of computerized simulations of a large number of learning decision-makers

⁶ Nagel explained that “reduction [...] is the explanation of a theory or a set of experimental laws established in one area of inquiry, by a theory usually though not invariably formulated for some other domain” (Nagel 1961). Reduction is therefore defined through the logical idea according to which a theory can be a definitional extension of another.

and it provides a specific way to study micro-foundations of the statistical regularities that emerge at the macro-level of economic systems (such as stylized facts)⁷. This approach provides a complementary perspective on the macro-patterns identified by the usual statistical models used by econophysicists. So doing, the latter extended their way of modelling the agents' behavior by enlarging the way they characterize the heterogeneity of individual components.

From the empirical side, the accuracy of the estimated quantities and/or of the statistical assumptions done for the modelling of economic and financial systems are typically model dependent and/or depending on the assumptions about the stochastic processes that are assumed to describe the evolution of the system of interest. The investigated system is always an open system which is monitored and described under specific modelling assumptions that are providing both the interpretative framework of the results obtained and the confidence intervals of the quantities estimated.

The traditional statistical care of Econometrics approaches of economic and financial systems has been progressively imported first in Econophysics studies and through them also in some more traditional area of studies of Physics. For example, the need for a more detailed statistical control of empirical analyses arises for physical systems not presenting manifest conservation laws as (i) interconnected complex systems well described by networked relations or (ii) evolving systems not characterized by a thermal equilibrium⁸.

Examples of this type of studies are seen in network science where statistical physicists single out specific links by considering (i) the rejection of a null hypothesis concerning the strength⁹ partitioning of a specific node (Serrano, Boguná, and Vespignani 2009, Radicchi, Ramasco, and Fortunato 2011) or by performing a statistical test of the over-expression or under-expression of repeated actions that can be modeled between elements of a bipartite system (Tumminello et al. 2011).

In fact, studies performed in Econophysics have shown that the analysis and modelling of physical systems can today start to overcome traditionally self-imposed limitations. Such situation ends up the strong bias of the discipline about the preference for studies of homogeneous systems at a thermal equilibrium and might foster interest towards studies of heterogeneous systems near or out of equilibrium presenting stationary or quasi-stationary statistical regularities. As examples, one can mention studies modelling the growth rate of firms or the evolution of wealth distribution described as a multiplicative stochastic process with constraints buildings (Levy, Solomon, and Ram 1996). The Gibrat's and Partetian law commonly used by the economic mainstream are clear examples of growth models reaching a stationary state that can model both economics and Physics open growing systems. The search for an explanation of the observed power law deviation from the growing lognormal Gibrat's law made clear that the presence of a boundary at low or zero value of the wealth or income is a key aspect

⁷ For further information about this increasing literature see Schinckus (2016).

⁸ It should be noted that the equilibrium discussed here is thermal equilibrium. The classic concept of equilibrium present in economic theory has an origin from Physics but it is a different concept of equilibrium being a form of equilibrium equivalent to the mechanical equilibrium observed in a system of masses and forces.

⁹ The strength of a node is the sum of the weights of links of the node.

for the observation of the power law deviation observed for high values of wealth as mathematically shown in the work of Kesten (1973).

2. 2. Signal detection in multivariate characterization of a complex system

The second example of the influence that Financial economics had on Physics concerns the signal detection in multivariate systems. This is a classic problem in Finance since the introduction of Markowitz portfolio optimization that pointed out the crucial role played by covariance matrix of assets' returns (Markowitz 1952). One key result of Econophysics concerns the use of random matrix theory (Laloux et al. 1999, Plerou et al. 2000, Conlon, Ruskin, and Crane 2009, Bouchaud and Potter 2011) and of filtering methods based on hierarchical clustering (Mantegna 1999) to detect signals (i.e. underlying regularities), which are present in correlation matrices estimated by using a finite number of records. The use of random matrices has given the opportunity to have an unsupervised criterion to discriminate between the information that is distinctively different from the one that is indistinguishable from a multivariate random process of given statistics.

In fact, the application of random matrix theory to the modelling of correlation matrices of stock returns has shown that correlation coefficients estimated using a finite number of records (a limitation that is always present in empirical estimations) present two basic distinct types of information. The first type is information that can be easily extracted from the structure of the correlation matrix. Random matrix theory makes clear the nature of this information. It is the information associated with the principal components whose spectral position lies outside the eigenvalues interval where random matrix theory predicts the presence of eigenvalues for random processes characterized as specific stochastic processes (in the most basic setting as independent Gaussian random processes). The second type of information is the information associated with eigenvalues falling inside the eigenvalues' interval predicted by random matrix theory. This type of information may or may not be associated with correlation coefficients estimated in a statistically reliable and unbiased way but the information associated has a nature that it is hardly distinguishable from a random pattern (Bun, Bouchaud, and Potters 2017). Therefore, extraction of this information can be achieved only with sophisticated filtering techniques (Tumminello, Lillo, and Mantegna 2010).

The use of random matrix theory in Econophysics triggered a large amount of activity in Physics dealing with the role of different underlying stochastic processes in the exact determination of the eigenvalues' spectrum. See, for example, the case of Levy processes (Cizeau and Bouchaud 1994, Burda et al. 2005, Arous and Guionnet 2008). Another wide area of investigation concerned the different methodologies to be used to extract the informative structure of the correlation matrix for both the two sets of information discussed above (Burda et al. 2004).

The need to extract sound information from the correlation matrix or, more generally, from any proximity matrix has motivated statistical physicists to propose several techniques that are successful in the filtering of information from a multivariate set. Examples are the extraction of the minimum spanning tree (Mantegna 1999) or of the planar maximally filtered graph (Tumminello et al. 2005) associated with a correlation

matrix or the planar maximally filtered graph associated with a partial correlation matrix.

The impact of finance in these studies considering the information content of time series (or more generally vectors) can also be seen by considering the progressive import, reconsideration and interpretation of Granger causality (Granger 1969) within the Physics community. Granger causality is a methodology originally developed in Finance that it is now used in many fields of research. Physics studies have shown that in the case of Gaussian variables it is equivalent to the more familiar concept of transfer entropy, a concept developed in the field of information theory (Barnett, Barrett, and Seth 2009).

This type of knowledge was originally used and discussed in the research performed in Econophysics but it is now knowledge of Statistical physics that can be used in any field of Physics where the multivariate nature of the system evolution is a key aspect of the system.

2. 3. The role of information in a complex adaptive system investigated in Physics

Our third and last example of the influence that Finance had in the setting of a Statistical physics problem and in fostering new concepts in this research area concerns a dynamical system with many agents that is presenting different phases in its dynamical evolution depending on a control parameter. The investigation of this type of system is the wide investigation of the so-called “minority game” and has a clear origin in a model originally proposed in economics.

Minority game (Challet and Zhang 1997) is a stylized version of the “El Farol bar” problem originally introduced by Arthur (1994). The “El Farol bar” problem is a well-known problem in game theory: a limited number of agents wish to take some action, but they will not benefit of the action if the majority of agents do the same. The motivation in economics was to introduce an illustrative example of the process of rational decision between two alternatives, says 0 and 1, of a group of rational agents in the presence of negative externalities. In this setting, there is no self-fulfilling equilibrium and therefore by assuming fully rational use of the public information the system oscillates between states that are always frustrating for the agents. By introducing his model, Arthur was able to show that a suboptimal (economic) equilibrium occurring at each time step around an a-priori optimal allocation of the resource is reached by the system by hypothesizing a bounded rational inductive reasoning of the agents.

By formalizing the “El Farol bar” model as a minority game, two econophysicists, Challet and Zhang (1997), inspired by financial analogies, defined a stylized model to be investigated and analyzed with tools and concepts of Statistical physics, particularly phase transition, control parameter, and order parameter. Minority game is a telling example how Econophysics provides a solution to a game theory problem that was created in economics. As Arthur Brian explained, “economists didn’t quite know what to make of [my paper presented at the January 1994 American Economic Association meeting]. My colleague at Santa Fe, Per Bak, did know however. He saw the

manuscript and began to fax it to his physics friends. The physics community took it up, and in the hands of Challet, Marsili and Zhang, it inspired something different than I expected — the Minority Game. El Farol emphasized (for me) the difficulties of formulating economic behavior in ill-defined problems. The Minority Game emphasizes something different: the efficiency of the solution” (Arthur 2004). The investigation and the results obtained were of great interest with many key concepts of Statistical physics observed in the stylized model and with key novelty about the order parameter. A phase transition is observed in the model between two distinct regimes of the deterministic time evolution of the system. More noteworthy, a quantity that may act as an order parameter of the phase transition (i.e. the quantity that is discriminating between the two distinct phases observed in the system) is directly expressed in terms of the information that can be extracted from the publicly available time series of the aggregated state of the system (that is the number of agents that decided state 0 or 1 at each time step, for instance to buy or to sell) (De Martino and Marsili 2006, Challet, Marsili, and Zhang 2013).

In other words, this is a stylized model of bounded rational heterogeneous agents that can be solved with state of the art tools and methods of Statistical physics. The solution obtained shows the existence of two distinct phases. The system will be in a given phase according to the value of the control parameter that in the simplest version of the game is $\alpha_c = 2^m/N$ where m is the number of records each agent uses to select the strategy to be used to make a choice and N is the number of agents participating to the game. The two states are different with respect to the ergodic or non-ergodic nature of the deterministic evolution. The phase with $\alpha < \alpha_c$ is non-ergodic and the global efficiency of the system is controlled by the initial conditions whereas the phase with $\alpha > \alpha_c$ is ergodic and information can be extracted by the agents from the publicly available information. The measure of the degree of predictability used in minority game is

$$H = \frac{1}{2^m} \sum_{\mu=1}^{2^m} E\{A(t)|\mu\}^2$$

where $A(t)$ is the measure of the global state of the systems fluctuating around zero and μ is the set of sequences used to define the strategy agents are using during the game. In the non-ergodic phase $E\{A(t)|\mu\} = 0$ for all μ and hence $H = 0$. In the other phase $H \neq 0$ and non-trivial predictions can be done about the $A(t)$ outcome. We have already noticed that that H acts like a ‘physical’ order parameter.

When initial conditions are set randomly the efficiency of the system, i.e. the amount of fluctuations $\sigma^2 = E\{A^2\}$ from the optimal allocation $A=0$ of choices is minimized when $\alpha \cong \alpha_c$ indicating that the control parameter α is useful to select the condition that maximize the overall suboptimal allocation of resources in the system.

Created in order to analyze an economic problem, improved by studying finance issues, minority game has been applied back to Physics and some related fields in order to model several problems. For instance, it is used in radio engineering and computer science in order to improve wireless networks (Mähönen and Petrova 2008, Sungwook 2014), secondary users battery life and network performance (Elmachkour, Daha, et al. 2014, Elmachkour, Sabir, et al. 2014), or to improve coordination in wireless sensor networks (Galstyan, Krishnamachari, and Lerman 2004). In computer science, minority game is used to improve the reconfigurable multi-core processors

(Shafique et al. 2011) or heterogeneous Delay Tolerant Networks (Sidi et al. 2013). It is also used in order to improve energy management system (EMS) of buildings (Zhang et al. 2012). According to Mähönen and Petrova (2008, 100), it could also be applied for studying the behaviors of flocks of birds.

It is worth noting that economics and finance are naturally dealing with systems where information is processed by rational agents and/or agents with bounded rationality, who are taking their decisions. By opposition, it is not the case in Physics. In Physics, while various forms of information are also investigated, the investigation is primarily limited to the use of information as a tool to quantify the degree of disorder present in the system. For example, in dynamical systems information production of a symbolic sequence associated with the time evolution of the dynamical system is describing whether the system is evolving in an attractor characterized by a simple structure or rather characterize by a complex structure as in the case of the so-called “strange attractors”.

Econophysics studies of the “minority game” have shown that studies of stylized physical system of economic, social or financial origin can investigate the use, spreading and aggregation of information in a way that is rigorous, explanatory and highly informative about the investigated system.

3. The key role of finance

The previous examples show that, while Econophysics at the beginning was driven by the application of Physics to finance, in the recent years the opposite influence has been observed. This section aims at identifying some distinctive characteristics of finance that explain the key role of this field in this new influence of Financial economics on Physics. We already mentioned the traditional statistical care of econometrics approaches. Two linked reasons are important: the increasing number of financial data and the methodology used by econophysicists.

3.1. The increasing number of financial data.

One distinctive characteristic of finance is the numerous available data, which has played a key role in the discovery of new phenomena and in scientific developments, and particularly in Physics. One well-known example is Louis Bachelier (1900) who was trained in mathematical physics. Bachelier's work is generally mentioned in order to show how Physics influences finance, particularly by proposing the first mathematical model for pricing Premium contract (called “Prime”), which was a conditional forward contract close to option. However, it is only the second step of Bachelier's reasoning. The first step shows, on contrary, the influence of financial data on Physics (Jovanovic 2012) that allowed Bachelier to introduce the continuous-time probabilities, the theory of Brownian motion, to develop the mathematical theory of diffusion (trajectories of Brownian motion), and to solve the parabolic diffusion equation five years before Albert Einstein (1905). Financial data was the major starting points of Bachelier and the foundations of all of his demonstrations. Precisely, the numerous available financial data gave to him the opportunity to demonstrate the equivalence between results obtained in discrete time and in continuous time.

What was true for Bachelier is still true today. Financial databases are nowadays the largest bases for social phenomena due to the progressive automation of financial markets. Precisely, finance was the first research area of economics where large amount of digitized data started to be stored, processed and analyzed. Since the end of the 1970s, all the major financial markets have been progressively automated thanks to computers¹⁰. In addition, some markets, like the foreign exchange market, became active 24 hours a day with electronic trading. Automation has allowed all transactions and all prices quoted to be recorded, leading to storage of a huge amount of financial data. Moreover, since the 1990s, use of computers has enabled the development of high-frequency transactions, and therefore the creation of high-frequency data (also called “intraday” data)¹¹. Previously, statistical data on financial markets were generally made up of a single value per day obtained by the average price or the last quotation of the day. Nowadays, with the recording of “intraday data,” all prices quoted and tens of thousands of transactions are conserved every single day (Engle and Russell 2004).

Due to the computerization of finance and the automation of financial markets, Financial economics becomes a discipline that produces a high rate of scientific data and information. Although finance is not the only one discipline in this case, the explosion of financial data, which has no equivalent in other social sciences fields, comes closer to the standards to which statistical physicists generally work. Precisely, in an economic system, one initial work was limited to analyzing time series comprising of order of magnitude 10^3 terms, and nowadays with high-frequency data the standard, one may have 10^8 terms, by comparison, macroscopic samples in physical systems that contain a huge number of interacting subunits, as many as Avogadro’s number, 6×10^{23} (Stanley and Plerou 2001, 563-4). Consequently, Physics is no more the discipline that is producing the more empirical data, and econophysicists interested in finance have new opportunities for discovering new phenomena and regularities. The increasing quantities of data, the introduction of intraday data, and the computerization of financial markets led to notable changes in techniques for detecting new phenomena (cf. Power law, etc.). Intraday data brought to light new phenomena that could not be detected or did not exist with monthly or daily data. Among these are strategic behaviors that influence price variations (Jovanovic and Schinckus 2017, 61).

It is worth mentioning that, by opposition to financial data, data traditionally used in Physics are mainly simulated data. Indeed, in Physics experience is highly controlled experience. Due to the numerous non-simulated data from financial markets, the detection of new phenomena is higher in financial data than in physical data. It provides the opportunity for econophysicists to obtain new results and develop models based on statistical physics.

¹⁰ In 1977, the Toronto Stock Exchange became the first stock exchange to be fully automated. Then, the Paris Stock Exchange (now Euronext) imported Toronto’s system and became fully automated at the end of the 1980s. These changes occurred for NASDAQ between 1994 and 2004, and later for the NYSE in 2006 with the introduction of the NYSE hybrid market. The Tokyo Stock Exchange switched to electronic trading for all transactions in 1999.

¹¹ High frequency trading, which is based on high-frequency data was virtually unknown ten years ago, yet it is estimated that high frequency traders in the USA nowadays participate in 70% or more of trades in equities and futures markets.

3.2. Methodological considerations

Methodological debates emerging in Econophysics led physicists to acknowledge that dealing with financial complex systems contributed to a better modelling of their field. In a recent TED talk, a worldwide recognized econophysicist, Didier Sornette (2013), claimed that financial bubbles are “everywhere even in the size of the planets”. This statement is intriguing and suggests that physicists might try to conceptualize physical phenomena by integrating elements they faced with in economics and finance. Surprisingly, many of econophysicists’ argumentations have some methodological similarities with practices used in Financial economics. Such perspective can methodologically be explained as the analogical export of a field outside of its borders is the most creative way to generate knowledge.

Econophysics has been developed by physicists who applied their methods to economic and finance data. So doing, they went out of their discipline and they cannot avoid to face with judgment of economists willing to protect their “disciplinary territory”. Although economists acknowledge the technical knowhow of econophysicists, they are reluctant with such kind of research. These disagreements are rooted in a set of communal cognitive values and tools that shape the foundations of scientific justification in both communities. As explained in Jovanovic and Schinckus (2017), these foundations are read and understood differently in the two disciplinary contexts.

When econophysicists extended their models in finance and economics they also implicitly export the scientific fabric usually associated with this model. For econophysicists, the epistemological justification of their works is quite simple: they used a familiar theoretical framework to describe a complex phenomenon that all exhibits the same key features to be studied through this frame. In other words, econophysicists did not produce their models out of nowhere: given specific characteristics (for instance, the emergence of extreme values in a particular dynamics) that they observe as physicists, they choose what appears for them an appropriate model (for instance, self-criticality theory) to describe this phenomenon. This approach is justified in two ways: by scientific foundations of this familiar framework and by the empirical adequacy of results. Such extension of Physics to other context is implicitly based on a justification that is internally (disciplinary) warranted but that can be questioned by scholars who are not familiar with Physics.

The way econophysicists have applied their knowledge to economics and finance is a telling example of a Duhemian use of analogy. Pierre Duhem (1861-1916) was a French physicist and philosopher well known for his works on the “Newtonian” (inductive) and the “Cartesian” methods (Ariew 2014). Although the notion of analogy is not ubiquitous in Duhem’s works, he referred to this concept when he wrote about how Physics as a field can evolve. More precisely, he explained that “The history of physics shows us that the search for analogies between two distinct categories of phenomena has perhaps been the surest and most fruitful method of all the procedures put in play in the constructions of physical theories” (Duhem [1914] 1962, 95). The French physicists illustrated his claim with a study on the Maxwell’s analogy between electrical flow and heat where he considered analogies as a final relationship between phenomena and theoretical treatment of phenomena. Precisely, “it may happen that the equations in which one of the theories is formulated is algebraically

identical to the equation expressing the other [...] [analogies are] intellectual economy, a method of discovery by associating two abstract systems; either one of them already known or both being formulated, they clarify each other" (Duhem [1914] 1962, 96-97).

This reasoning *per analogiam* is also presented by Duhem as a way of understanding science as a human activity developing in time and requiring transgressions across the borders of the domain under investigation (Duhem [1914] 1962, 95): the development of Econophysics seems to result from such way of defining scientific activity. According to Duhem, scientists are not free in their choice of assumptions or models at a given time. Scientific knowledge, experience and even scientists' common sense are always somewhat related to a specific tradition. In this sense, theories of the past act as the "nuclei of the victorious theories of the future" (Schafer 2006). In other words, the analogical extension of knowledge is always constrained by a particular conceptual framework in which what is observed and how this thing is observed cannot be totally separated (Duhem [1914] 1962). Such perspective is interesting because it offers a mode of transfer for analogies. The justification of this transfer of this formal analogy from Physics to Economics and Finance can be found in a Duhemian analysis to understand what happens in the econophysicists' mind. By applying their models and concepts in Economics and Finance, econophysicists gradually and analogically extended the epistemic domain of these well-known models and concepts in line with Duhemian use of analogy. What is specifically Duhemian in the formal analogies proposed by econophysicists is the way these scientists conjointly extend the analogical properties and the theoretical framework justifying these properties to Economics and finance. Analogies (and their consequences), like assumptions, cannot be formulated in isolation of a peculiar theoretical frame that supports them. Duhem ([1914] 1962) explained that this kind of extension does not pop up from nowhere as the result of scholars' individual arbitrariness but it rather results from the gradual development of a logic belonging to a specific tradition. Regarding to this aspect,

"Reasoning by analogy has to start with previous knowledge. It has to rely on ideas that are familiar and have proved to be useful in a particular field of research. These ideas are, then, *per analogiam*, carried over in a new domain. Applying familiar ideas to new domain implies usually modifications in the inherited body of knowledge; every genuine development of science does not only add new materials to former knowledge but does single out certain sections as no longer tenable. New knowledge, if new it is, will negate some part of other if the received knowledge" (Schafer 2006, 80).

This Duhemian use of analogy has some epistemological consequences, as Schafer explained it,

"this [Duhem] reconstruction of physics required the strict abolition of explanatory ambition [...] and restriction to the descriptive function of physical theory. According to this, the only appraisal of physical theory that could claim to be rational consisted in the check of empirical adequacy which is restricted to the purely internal context of justification" (Schafer 2006, 84).

In this perspective, Econophysics can be perceived as a simple analogy but rather as a justified new way of dealing with financial/economic systems. This situation explains

why econophysicists consider they could replace (or they are totally indifferent to) the existing economic knowledge. Such Duhemian way of dealing with an imported analogy as a replacement of existing knowledge clarifies how econophysicists bring their reasoning in Economics and finance. First of all, Duhem acknowledged that mathematical structure of a model as the core of Physics – precisely, he considered that “a physical theory is a system of mathematical propositions, deduced from a small number of principles which aim to represent as simply as completely and exactly as possible a set of experimental laws” (Duhem [1914] 1962, 9). So doing, Duhem emphasized the dominance of the mathematical deductive method in Physics. By combining the Duhemian use of analogy in the extension of knowledge and the importance this deductive reasoning, we can illustrate this analogical reasoning with the Physics statement according to which a power law, which is a key characteristic of Econophysics’ models, is an expression of self-criticality (Bak 1994).

Statement 1: Complex phenomena are composed by a high number of interacting micro-elements that generate a dynamic that can be described by a power-law.

Statement 2: Financial markets\economic systems are complex phenomena.

Conclusion: Financial markets\economic systems exhibit power laws.

Interestingly, such analogical reasoning extending physical concepts into finance generates un-thought situations simply because financial systems and physical ones are materially very different. In this context, unknown (new) aspects must be formalized and integrated into knowledge through a process in which analogies play an important heuristic role in the way of conceptualizing the unknown. Examples mentioned in the previous section illustrated such way of developing knowledge. This way of developing Physics is in accordance with Duhem’s approach in which only abstract and general principles (experimental law) can guide the scholars’ mind in unknown situations. This approach is then justified in three ways: 1) the familiarity/scientific foundations of the imported framework, 2) the empirical adequacy of results (the observation of statistical patterns in physical and economic/financial systems) and, 3) extension of knowledge in Physics. In a Duhemian perspective, this analogical extension of Physics is justified for econophysicists only because the internal logic of their field is respected.

The role of analogies explains that from a physicist’s point of view, Econophysics can be perceived as an analogical and idealized extension of Physics models, tools and concepts that appear to be theoretically, empirically and logically justified. This explains, from a methodological perspective, how econophysicists can justify the transfers from Physics to Finance. Interestingly, this analogical extension of Physics would not be possible without the specificities of the field in which Econophysics has mainly been developed: finance. Analogies are not necessary unilaterally implemented when, for instance, a formalization of the system 1 is used to describe the system 2 ($S1 \rightarrow S2$) simply because the perfect synonymy between two systems is impossible. Consequently, such situation generates some gaps paving the way to a reciprocal influence between the two systems associated in the analogies ($S1 \leftrightarrow S2$).

Giorgio Israel (1996) emphasized such reciprocal influences when he worked on the importance of “mathematical analogies” in science. These analogies are based on the existence of unifying mathematical simple models that are not dedicated to the phenomena studied. Mathematical modeling then uses mathematical analogies by

means of which the same mathematical formalism is able to account for heterogeneous phenomena like those in finance and in Physics. These heterogeneous phenomena are “only interconnected by an analogy that is expressed in the form of a common mathematical description” (Israel 1996, 41). The model then is an effective reproduction of reality without ontology, one that may provide an explanation of phenomena. Mathematical analogies illustrate the transfers from Physics to Finance, allowing the creation of Econophysics and the extension of knowledge in Physics. In the same vein, these mathematical analogies able also the same kind of transfers but from Finance to Physics by suggesting some unknown aspects that physicists gradually integrated into their disciplinary knowledge.

4. Conclusion

By opposition to the traditional influence of Physics on Financial economics, and more generally on Economics, the mathematical analogies and the digitalization of financial data have created a context that allows a mutual influence. The Financial economy, and more generally the economy, is no longer considered an application field where methods, models and tools from Physics can be used. Econophysicists can use results from Financial economics in order to explore new challenges in Physics. Even though recent, this methodological trend can be observed in Physics – this paper aims at introducing this moving nature of knowledge between Physics and Financial economics

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